Narrowband Jammer Suppression Using an Adaptive Filter and Temporal Root Tracking

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Abstract— **This paper presents an adaptive algorithm using the normalized LMS algorithm and a prediction-error filter to suppress high-SNR stationary narrowband jammers corrupting a composite input signal. Any broadband signals present in the input signal are preserved using the proposed technique. A combination of adaptive filter root tracking and notch filtering allows precise determination of the jammer frequencies while leaving the remainder of the input signal untouched. An illustrative simulation using synthetic data is also provided to support the proposed algorithm.**

I. Introduction

The algorithm in this paper isolates stationary narrowband sinusoidal interferers using an adaptive transversal filter configured as a one-step linear predictor. The adaptive filter weights are updated using the well-known normalized LMS algorithm. By using the normalized LMS algorithm, the problem of tracking the roots temporally is mitigated. Upon detecting any strong sinusoidal interference, the resulting interfering frequencies are suppressed using standard parametric notch filters. The proposed approach is appropriate for several signal processing scenarios including a separable space-time adaptive processing (STAP) algorithm, and the removal of sinusoidal interference from a corrupted broadband signal.

II. Linear Prediction via Adaptive Filtering

In this paper, the input signal is assumed to contain a broadband signal with M narrowband interferers embedded in white noise. Let the input signal be defined as

$$x(n) = d(n) + \sum_{k=1}^{M} J_k(n) + u(n) \tag{1}$$

where $d(n)$ is the desired broadband signal, $J_k(n)$ represents each narrowband interferer, and $u(n)$ is zero-mean white noise.

The instantaneous measurement of the narrowband jammer frequencies are determined using an adaptive transversal filter configured as a one-step linear predictor, also known as a prediction-error filter (PEF). One-step linear prediction attempts to predict the next value of a signal using a finite number of past samples p

$$x(n) = \sum_{k=1}^{p} w(k)x(n-k) \tag{2}$$

A prediction-error filter attempts to whiten, or flatten, the spectrum of the incoming signal. Thus, the PEF will attack narrowband interference as well as any broadband signal in its native operation. Given stationary input signals, the PEF updates the adaptive filter weights in a fashion that converges to the optimal Wiener solution. Furthermore, the optimal filter weights are related to the autoregressive (AR) spectral estimate coefficients such that, $a(k) = -w_{opt}(k)$, $k = 1, 2, ..., p$. By relating the two processes, the adaptive PEF transfer function $H(z)$ is related to the AR model polynomial $A(z)$ as follows:

$$H(z) = 1 - \sum_{k=1}^{p} w_{opt}(k)\ z^{-k} \tag{3}$$

$$= 1 + \sum_{k=1}^{p} a(k) z^{-k} = A(z) \tag{4}$$

A. PEF Transfer Function Root Characteristics

Since $A(z)$ is directly related to $H(z)$, the findings in [4] can be applied to the PEF. Specifically, Kay [4] states that for a sinusoid (AR process) embedded in white noise, increasing the model order p increases the radius of the sinusoidal root on the unit circle. However, for a constant signal-to-noise ratio (SNR), the model order cannot be increased indefinitely. The noise roots will begin to approach the unit circle and ultimately result in spurious noise peaks. Likewise, for decreasing SNR, the radius of the sinusoidal root decreases until it is equal to the radius of the noise roots.

Furthermore, Kay [4] states that any extra noise roots that model the white noise tend to be uniformly distributed in angle around the unit circle. For high-SNR situations, these noise roots reside far enough away from the unit circle so that in a scenario such as jamming or interference, the PEF transfer function roots that correspond to each narrowband jammer will reside near the unit circle while the noise and broadband roots will reside elsewhere. This fact is used to effectively classify sinusoidal roots from all other roots in the composite input signal.

B. Model Order Selection

It is shown in [4] that increasing the model order for a given SNR also reduces the frequency bias of the sinusoidal roots. Using an appropriate model order p, which corresponds to the number of adaptive filter weights, allows for better frequency determination of the narrowband jammers. A large enough model order must be carefully chosen so that there are enough degrees of freedom to attenuate M narrowband jammers, but also small enough so as not to introduce spurious noise roots. Choosing an appropriate model order for an AR process is well documented in [1], [3], and [10].

III. Normalized Least Mean Square (NLMS)

The NLMS algorithm is a variant to the well-known LMS algorithm. The NLMS algorithm updates the adaptive filter weights with a gain factor that is inversely proportional to the input signal power in the adaptive filter taps

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\beta}{\|\mathbf{x}(n)\|^2} e(n)\mathbf{x}(n) \quad (5)$$

For a PEF, vectors $\mathbf{x}(n)$ and $\mathbf{w}(n)$ in Eq. (5) are defined as

$$\mathbf{x}(n) = [x(n-1)\ x(n-2)\ ...\ x(n-p)]^T \quad (6)$$

$$\mathbf{w}(n) = [w_1(n)\ w_2(n)\ ...\ w_p(n)]^T \quad (7)$$

where p is the number of transversal filter taps and β is the normalized step size. The error term $e(n)$ in Eq. (5) is defined as

$$e(n) = x(n) - \mathbf{w}^H(n)\mathbf{x}(n) \quad (8)$$

The NLMS algorithm has been proven to be the solution to a constrained optimization problem [2]. In fact, Haykin [2] shows that the NLMS algorithm updates the adaptive weights $\mathbf{w}(n+1)$ in a fashion that exhibits minimal change with respect to the current weights, $\mathbf{w}(n)$, in the Euclidean norm sense. Thus, from one iteration to the next, the root locations are perturbed minimally with respect to the squared Euclidean distance. Conceptualizing the normalized LMS algorithm in this manner is paramount, as this characteristic of NLMS alleviates the problem of tracking the roots of the PEF transfer function over time.

IV. Root Tracking

The root tracking algorithm determines the trajectories of the adaptive PEF roots. Finding the roots of Eq. (3) at iteration n yields the roots of PEF transfer function $H(z)$. The roots of $H(z)$ at iteration n can be written in polynomial form

$$D_n(z) = \prod_{i=0}^{p-1}(1 - z^{-1}\phi_i(n)) \quad (9)$$

The roots themselves at iteration n can also be expressed in vector form

$$\mathbf{r}(n) = [\phi_0(n)\ \phi_1(n)\ ...\ \phi_{p-1}(n)]^T \quad (10)$$

By utilizing the normalized LMS algorithm, associating the root locations at iteration n to the root locations at iteration $n-1$ is achieved by defining a Euclidean norm cost function $J(n)$. For every root $\phi_i(n)$, $i = 0...p-1$ at the current time n, the cost function $J(n)$ is minimized

$$J(n) = \|\phi_i(n) - \phi_j(n-1)\|^2 \quad (11)$$

For each root $\phi_i(n)$ at time n, Eq. (11) is minimized with respect to the roots $\phi_j(n-1)$ at time $n-1$. The previous root $\phi_j(n-1)$ that minimizes $J(n)$ at time n is hypothesized to be the same root, only that it has moved by the cost distance $J(n)$. This is believed to be a manifestation of the NLMS algorithm described previously. Thus, the movement of each root about the complex plane can be stored into root trajectories and tracked temporally.

V. Determination of Sinusoidal Roots

By tracking the PEF roots, certain statistics can be calculated over time to determine which roots are indicative of strong sinusoidal interference. In this paper, a simplistic but sufficient set of judicious calculations are used to determine which roots are characteristic of high-SNR narrowband jammers.

A circular boundary of radius $\tau \leq 1$ on the complex plane serves as a simple discriminator between the sinusoidal roots and the remaining broadband and noise roots. Any root $\phi_i(n)$ with a magnitude greater than τ is assigned a persistence statistic $\kappa(n)$, which is defined by a constant-gain alpha filter. Since $D_n(z)$ in Eq. (9) can be written as a $(N \times p)$ matrix of adaptive filter roots with n representing the temporal variable and N the number of iterations, define each root's persistence as

$$\kappa_i(n) = (1-\alpha) \cdot \kappa_i(n-1) + \alpha \cdot b_i(n) \tag{12}$$

where $\alpha \leq 1$ and

$$b_i(n) = \begin{cases} 1, \text{ if } |\phi_i(n)| \geq \tau \\ 0, \text{ otherwise} \end{cases} \tag{13}$$

The alpha filter $\kappa_i(n)$ in Eq. (12) is a single-tap IIR filter with an exponential impulse response related to α. Eq. (12) has a transfer function equal to

$$H_{alpha}(z) = \frac{\alpha}{1-(1-\alpha)z^{-1}} \tag{14}$$

During the convergence time of the adaptive PEF, the persistence statistic is calculated for each root at every iteration. Upon stopping the adaptation of the PEF at iteration $N_{\max}$, any root ϕ_i with a persistence statistic that surpasses a minimum persistence is deemed a sinusoidal root:

If $\kappa_i(N_{\max}) \geq \rho_{\min}$
ϕ_i is a sinusoidal root

From the previous statements, the persistence criteria is justifiable because high-SNR sinusoidal roots migrate very close to the unit circle while noise and broadband roots situate themselves farther away from the unit circle.

The sinusoidal roots are also perturbed from their optimal frequency due to use of the NLMS algorithm. Thus, the estimate of each narrowband jammer frequency is taken to be the average of each root location ϕ_i that surpasses the persistence criteria

$$\widehat{\phi_{opt}} = \frac{1}{L} \sum_{k=L}^{2L-1} \phi_i(k) \tag{15}$$

In Eq. (15), L is a chosen starting iteration where $\kappa_i(L) \geq \rho_{\min}$. Of course, if the input in Eq. (1) is a real-valued signal, the roots will appear in complex-conjugate pairs. In this case, only one of the $\widehat{\phi_{opt}}$ sinusoidal roots from each conjugate pair needs to be retained to determine its frequency.

VI. Parametric Notch Filtering

Once the sinusoidal roots are excised from the remaining PEF transfer function roots, the corresponding jammer frequencies can be suppressed using standard notching techniques. The approach proposed in this paper involves building a set of parametric notch filters using the excised sinusoidal roots $\widehat{\phi_{opt}}$ from the root tracking algorithm. The center frequencies for the notch filters correspond to the located jammer frequencies.

A parametric notch filter is a second-order IIR filter that places a single notch at a given frequency while maintaining a constant gain for all other frequencies. So, only the offending jammer is attenuated, preserving the rest of the frequency spectrum. The response of each parametric notch filter (PNF) is [5]

$$H_{pnf}(z) = \frac{1 - 2 \cdot \cos\omega_0 \cdot z^{-1} + z^{-2}}{1 - 2R \cdot \cos\omega_0 \cdot z^{-1} + R^2 \cdot z^{-2}} \tag{16}$$

where R is the radius of the filter poles from the origin. The parametric notch filter places two zeros on the unit circle and two poles a distance of $(1-R)$ behind the zeros. The distance the poles are from the unit circle defines the amount of attenuation. The closer R is to the unity, the more attenuation. The distance $(1-R)$ also governs the 3-dB cutoff frequency of the filter. The closer R is to unity, the sharper the notch, but the slower the impulse response of the filter. A rule of thumb to estimate the 3-dB digital frequency for a parametric filter when $R \lesssim 1$ is [5]

$$\omega_{3dB} \simeq 2(1-R) \tag{17}$$

If the root tracking algorithm detects M narrowband jammer frequencies, then M parametric filters are generated and cascaded together. The overall notch filter that attenuates all the sinusoidal interferers can be written as

$$H_{total}(z) = \prod_{k=0}^{M-1} H_{pnf_k}(z) \tag{18}$$

After a convergence time of $N_{\max}$ iterations is achieved, the notch filter defined by Eq. (18) is constructed and begins to filter the corrupted input signal.

VII. Simulation Results

An illustrative simulation was carried out using synthetic data in order to demonstrate the capability of the proposed algorithm.

Fig. 1 shows a representative case of a corrupted real-valued input signal. The desired broadband

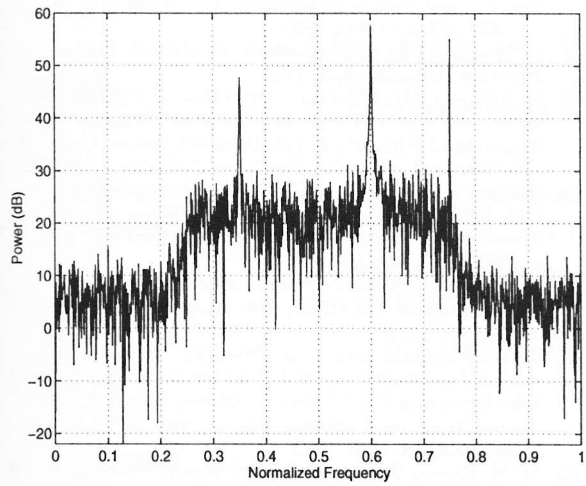

Fig. 1. Periodogram of input broadband signal corrupted by three narrowband jammers.

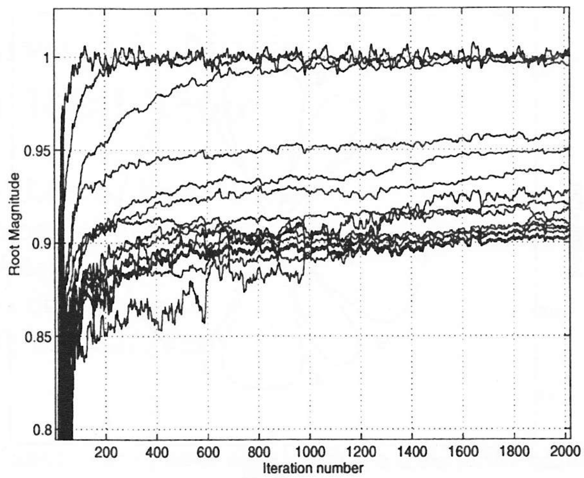

Fig. 2. Magnitude of root trajectories for filter roots with positive angles

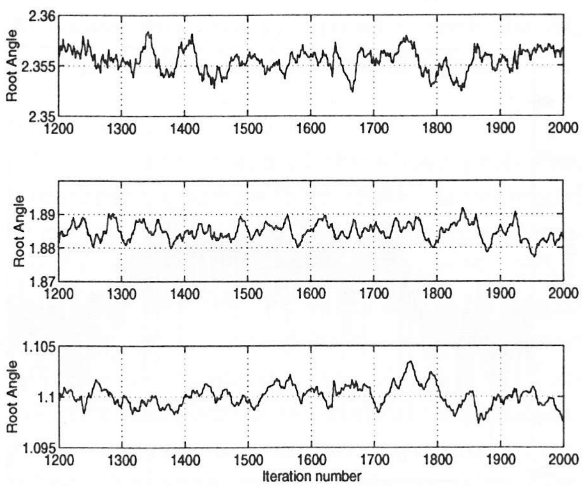

Fig. 3. Perturbation of detected sinusoid root angles over time.

signal was corrupted by three strong real-valued narrowband jammers. The three jammers were strategically placed at normalized frequencies of $f = 0.35$, 0.6, and 0.75. The jammer-to-noise power (J/N) ratio of each jammer was 20, 30, and 25 dB, respectively. The PEF was configured with $p = 32$ adaptive filter weights using the NLMS algorithm with $\beta = 0.1$.

The root tracking algorithm was initialized with the following parameters: $\tau = 0.99$, $\rho_{\min} = 0.9$, and $\alpha = 0.2$. Since the input signal was real, the root trajectories for roots that have positive angles are shown in Fig. 2. It is easily observable in Fig. 2 that the roots corresponding to the narrowband jammers reside very close to the unit circle and have near unity magnitude.

The PEF was stopped after it converged for a sufficient amount of time, and the root tracking algorithm returned three candidate roots indicative of strong sinusoids. The angle jitter of each candidate root is apparent in Fig. 3. This jitter is due to the use of the NLMS algorithm, demonstrating why Eq. (15) is necessary to extract an estimate of the jammer frequencies.

After applying Eq. (15), the detected sinusoids were located at normalized frequencies of $\widehat{f} = 0.3502$, 0.6, and 0.7497. Obviously, these detected frequencies are close to the actual frequencies of the jammers. The parametric notch filtering stage was completed by constructing three parametric notch filters using $R = 0.95$ and $\omega_0 = 2\pi\widehat{f}$ as the center frequencies. The resulting pole-zero plot along with the composite polar magnitude parametric notch filter frequency response is shown in Fig. 4.

The final step was to subject the corrupted input signal to the cascaded parametric notch filters. The power spectral density of the filtered input signal is shown in Fig. 5.

Fig. 5 shows that the narrowband jammers are sufficiently attenuated while leaving the remainder of the input signal intact.

VIII. Conclusion

The hybrid method of root tracking using a PEF and notch filtering proves that stationary narrowband jammers can effectively be removed from a mixed-signal environment. Further research is required to determine if this technique is capable

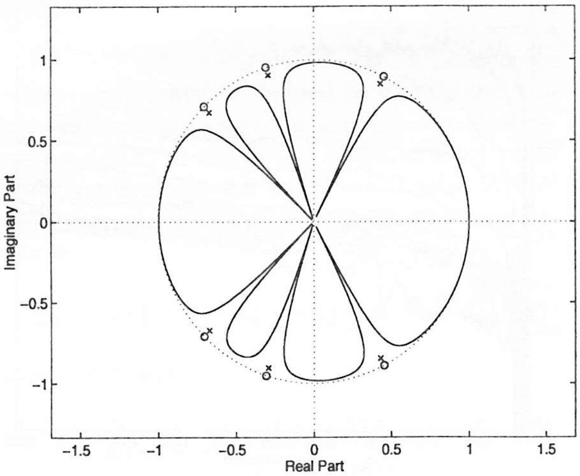

Fig. 4. Location of parametric notch filter poles and zeros with R = 0.95. The normalized polar magnitude filter response is also overlayed.

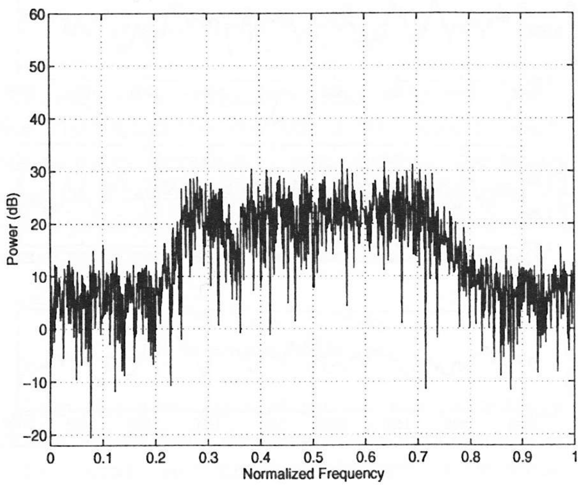

Fig. 5. Periodogram of input signal after suppression of the narrowband jammers.

of tracking nonstationary jammers in a real-time system. Additional analysis is planned to see if this method can combat narrowband jammers in a spread-spectrum system utilizing spatiotemporal signal processing techniques.

REFERENCES

[1] S. M. Kay and S. L. Marple, "Spectrum Analysis-A Modern Perspective", Proceedings of the IEEE, vol. 69, No. 11, pp. 15-49, November 1981.

[2] S. Haykin, *Adaptive Filter Theory*, Third Edition, Prentice Hall, 1996.

[3] S. M. Kay, *Modern Spectral Estimation*, Prentice Hall, 1988.

[4] S. M. Kay, "The Effects of Noise on the Autoregressive Spectral Estimator", IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. ASSP-27, No. 5, pp. 478-485, October 1979.

[5] S. J. Orfanidis, *Introduction to Signal Processing*, Prentice Hall, Ch. 6, 8, 1996.

[6] L. Li and L. B. Milstein, "Rejection of Narrow-Band Interference in PN Spread-Spectrum Systems Using Transversal Filters", IEEE Transactions on Communications, vol. COM-30, No. 5, pp. 925-928, May 1982.

[7] S. Kay, "Noise Compensation for Autoregressive Spectral Estimates", IEEE Transactions on Acoustics, Speech, and Signal Processing, vol. ASSP-28, No. 3, pp.292-303, June 1980.

[8] J. Markhoul, "Linear Prediction: A Tutorial Review", Proceedings of the IEEE, vol. 63, pp. 561-580, April 1975.

[9] J. W. Ketchum and J. G. Proakis, "Adaptive Algorithms for Estimating and Suppressing Narrow-Band Interference in PN Spread-Spectrum Systems", IEEE Transactions on Communications, vol. COM-30, No. 5, pp. 913-924, May 1982.

[10] M. H. Hayes, *Statistical Digital Signal Processing and Modeling*, John Wiley & Sons, Inc., 1996.

[11] F. M. Hsu and A. A. Giordano, "Digital Whitening Techniques for Improving Spread Spectrum Communications Performance in the Presence of Narrowband Jamming and Interference", IEEE Transactions on Communications, vol. COM-26, No. 2, pp. 209-216, February 1978.