# Intro to USCG DGPS "BEACON" Signal

## Undersampling ADCs for Implementing a MSK Demodulator

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This document outlines some important algorithmic, software, and hardware portions of the beacon DGPS system. The purpose of this document was to collect as much information on this GPS subsystem in order to allow other engineers to learn about the Beacon signal structure.

### BEACON SYSTEM

The beacon system contains RF hardware, DSP firmware, and general processor firmware. The sampled RF beacon signal is fed into an ADC, which is read directly by the DSP. The DSP handles all signal processing, demodulation, and data decoding. The DSP is controlled by the host processor. The processor has the responsibility of loading different DSP programs, maintaining beacon databases, and other housekeeping issues related to tracking beacon signals. The DSP strictly handles low level data demodulation, and it shares its data in a shared RAM configuration with the host processor.

### BEACON RF

The original beacon signal band resides from 275-325kHz. It was modified by the USCG to a band of 283.5-325kHz. The RF antenna pipes this signal to a beacon analog filter. The filter is a bandpass structure of LC tuned circuits residing in the beacon signal path. This filter is tuned to the beacon signal band from 283.5-325kHz. This acts as the beacon anti-aliasing filter before sampling.

### BEACON SAMPLING

After the analog filter, the beacon signal is sampled via a 12-bit ADC. This gives about 6dB/bit*12bits = 72dB of dynamic range. The bandwidth of the beacon signal is 325-283.5 = 41.5kHz. The sampling frequency used in the DSP is Fs = 173kHz. We are effectively undersampling the RF signal, but we still abide by the Nyquist rate since the signal bandwidth $B < Fs/2$. The undersampling operation is known as "constructive aliasing", which results in a "free mix" of the beacon RF signal to a lower IF frequency band. This is commonly done in RF schemes to perform a free mixdown without having to use expensive analog oscillators.

The sampling scheme is best illustrated below. Here, Fs = 173kHz (=25e6/9/16). The graph shows an arbitrary real frequency response in the beacon frequency band. The analog BPF filter suppresses everything else outside this band before sampling. So, for a real signal, we have a symmetric magnitude response from 275-325kHz. The continuous-time frequency spectrum of the beacon signal is denoted by $X_c(f)$.

You can also see in Fig. 1 that the original beacon frequency spectrum falls in between the frequencies 3fs/2 (260.4kHz) and 2fs (347.2kHz).

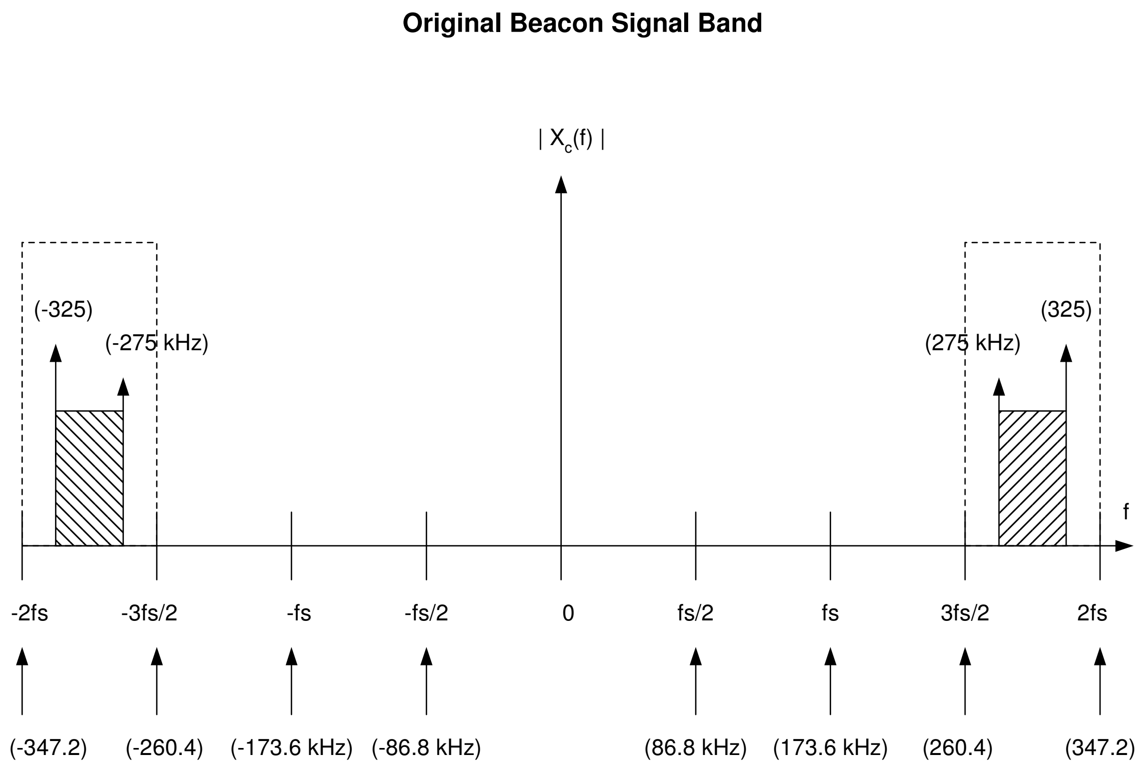


**Fig. 1 - Fourier Transform (Spectrum) of the Original Beacon Signal**

When we sample the beacon signal, the resulting sampled signal has replicated spectral components at multiples of the sample frequency Fs. This is the fundamental concept in a digital sampling system. In this frequency domain representation of sampling, this means that the spectrum in Fig. 1 is shifted by nFs, where n =…-3,-2,-1,0,1,2,3,… and then added together. In other words, take the above Fig. 1, shift the beacon signal band by –3Fs, then add it to Fig. 1 again. Then shift by –2Fs, add it to the figure again. Do this for –Fs, Fs, 2Fs, 3Fs, etc to get the sampled frequency domain signal. The resulting frequency spectrum is a sum of the original frequency spectrum shifted in time. This new spectrum is denoted $X_s(f)$.

The representative equation for frequency domain sampling is:

$$X_s(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X_c(f - nf_s) \tag{1}$$

When you do this replication and add all of the frequency copies together, you get a plot as shown in Fig. 2:

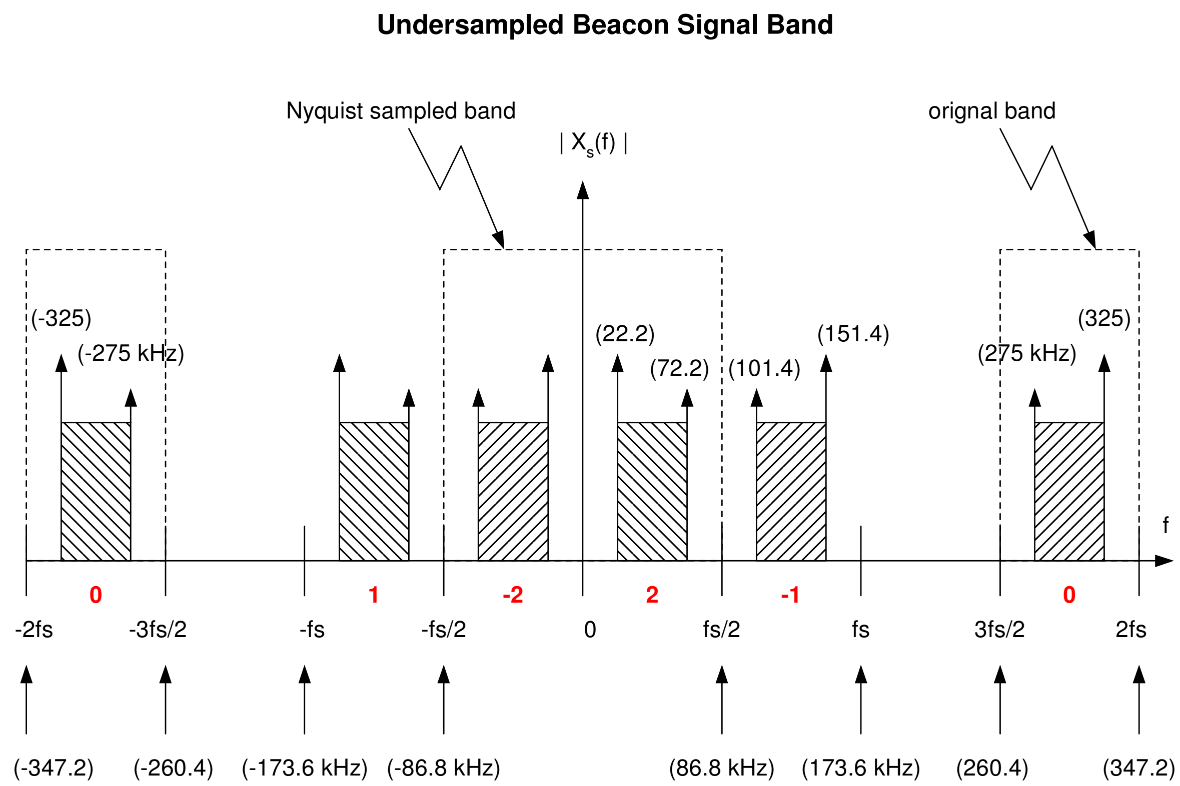


**Fig. 2 - Undersampled Beacon Frequency Spectrum**

In Fig. 2, the shifted replicas for +/-2Fs, +/-Fs, and 0 are shown. All other copies of the frequency spectrum lie outside of the plot and do not concern us. The red numbers shown in Fig. 2 denote the integer value "n" for each frequency-shifted copy. For n=0, we get the original spectrum (as shown in Fig. 1). For n=1, we have shifted the original spectrum by Fs to the right. For n=-1, we have shifted a copy of the original spectrum by Fs to the left. One can see that in the Nyquist band from f = [-fs/2, fs/2], the n=-2 and n=2 spectrum copies fall into our Nyquist sample band. This is what was referred to as the "free mix" property previously discussed. The original sample band from 275-325kHz has been mixed down to 22.2-72.2kHz.

Note the bandwidth of the signal is still the same, B = 50 kHz. This still holds since our original signal bandwidth B < fs/2. Since fs=173kHz, we are oversampling the actual signal content, even when it is located at a higher IF carrier frequency. This is the crux of the Nyquist sampling theory, which states that you must sample the signal at greater than twice the signal ***bandwidth***. This is not to be confused with sampling twice the highest frequency in a signal, which is often a mistaken concept.

So, the signal spectrum in Fig. 2 from frequencies [-fs/2 fs/2] is what the digital samples represent after coming out of the ADC. Another phenomenon you can see in Fig. 2 is that the spectrum is ***REVERSED*** from the original signal! That is, the positive spectrum at 275-325kHz is the mirror image of the positive spectrum at 22.2-72.2kHz. This is also an artifact of the undersampling operation. For the beacon signal, this effectively means that a sinusoid at frequency 275kHz maps down to 72.2kHz, whereas a sinusoid at frequency 325kHz maps down to 22.2kHz – resulting in a mirroring effect. The spectrum at [-fs/2 fs/2] can be reversed again with DSP algorithms, if this spectral reversal is unwanted. This is covered in the quadrature sampling section.

BEACON SIGNAL STRUCTURE

The beacon signal band consists of radiobeacon stations transmitting MSK data at a variety of bitrates (50,100,200bps). The beacon stations are spaced 1kHz apart on multiples of 1kHz (318kHz as opposed to 318.5kHz). With this station spacing, there can be a max of around 41 stations in the signal band (B = 41kHz/1kHz). As stated in the USCG document, all 200bps stations are centered at 283kHz + n*2kHz, where n is an integer from 0 to 21.

The beacon data is MSK modulated. MSK can be viewed as a continuous phase FSK signal. Since the phase of the signal no longer contains discontinuities during bit transitions, MSK has a much more efficient bandwidth than normal FSK.

The frequency spacing between the 2 MSK transmission frequencies is equal to ½ the bit rate. The modulation rates are chosen to assure phase continuity during bit transitions.

MSK PRIMER

The MSK signal can be viewed as a special case of OQPSK to understand its derivation. In QPSK, the incoming bit stream has bit periods equal to T. These bits are split into even and odd bit sequences. These new bit streams have bit periods of 2T (twice as long). These bit streams are modulated with sin and cos carriers to create an orthogonal waveform. QPSK can be written as:

$$s(t) = \frac{1}{\sqrt{2}} a_i(t)\cos(w_c t + \frac{\pi}{4}) + \frac{1}{\sqrt{2}} a_q(t)\sin(w_c t + \frac{\pi}{4}) \quad (2)$$

which takes on the following waveforms for the different bit combinations:

| $a_i(t)$, $a_q(t)$ | s (t) |
|---|---|
| (-1,-1) | $-\cos(w_c t)$ |
| (-1,1) | $\sin(w_c t)$ |
| (1,-1) | $-\sin(w_c t)$ |
| (1,1) | $\cos(w_c t)$ |

Again, $a_i(t)$ is the even bit stream and $a_q(t)$ is the odd bit stream. For valid even and odd bit combinations, we can draw a signal-space diagram for generated waveform. The bit combinations over 2T seconds are denoted as bit pairs (ai, aq).

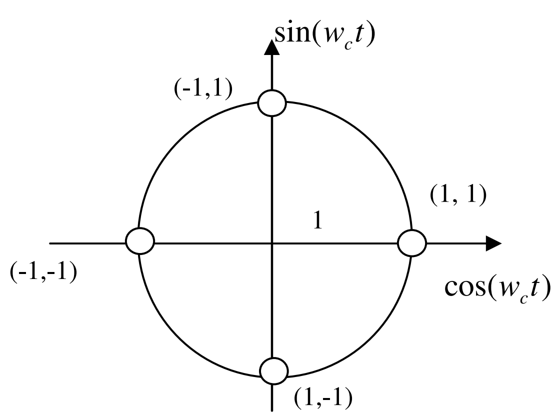


**Fig. 3 – QPSK signal-space diagram**

The above diagram shows that every bit combination over a 2T bit period takes on a discrete point in the 2D signal space. Moreover, since both bits can change during a bit transition, a 180° instantaneous phase change can occur during both bit transitions (i.e., when s(t) goes from cos(t) to –cos(t) in a bit transition). This causes much more harmonic content in the RF link, making QPSK not as bandwidth efficient as it could be.

In OQPSK, the data is multiplexed similarly as in QPSK, except that the odd stream is also staggered in time by T seconds with respect to the even stream. This multiplexing operation constrains the phase change in the carrier during bit transitions. This staggering of the bit streams constrains the maximum phase change to 90°, since now only one bit will change every T seconds. OQPSK has the same waveforms as QPSK, but since the phase change is minimized, the spectral efficiency of OQPSK is better than normal QPSK.

MSK is similar to OQPSK, with the difference that the bits are weighted with sinusoidal pulses rather than the normal rectangular pulses. An MSK signal can be written as:

$$s(t) = a_i(t)\cos(\frac{\pi t}{2T})\cos(w_c t) + a_q(t)\sin(\frac{\pi t}{2T})\sin(w_c t) \qquad (3)$$

which takes on the following waveforms for the different bit combinations:

| $a_i(t)$, $a_q(t)$ | S (t) |
|---|---|
| (-1,-1) | $-\cos(f_c t - t/4T)$ |
| (-1,1) | $-\cos(f_c t + t/4T)$ |
| (1,-1) | $\cos(f_c t + t/4T)$ |
| (1,1) | $\cos(f_c t - t/4T)$ |

The instantaneous frequency of the 2 different waveforms is:

$$f_i = \frac{d}{dt}(f_c t \pm \frac{t}{4T}) = f_c \pm \frac{1}{4T} \qquad (4)$$

$$\therefore f_+ = f_c + \frac{1}{4T}$$

$$\therefore f_- = f_c - \frac{1}{4T}$$

So, the MSK signal consists of 2 frequencies that deviate 1/4T hertz from the carrier frequency. Using the above definition, MSK can also be viewed as a CPFSK signal with a frequency separation equal to 1/2T, which is ½ the bit rate (where bit rate B=1/T). Since the phase of the above waveforms linearly increase or decrease in time, the phase of the MSK signal can be made continuous at the bit transitions by choosing the carrier frequency equal to an integer multiple of B/4 (1/4 the bit rate).

In the beacon signal structure, a binary “0” is defined as a 90° phase retardation from the carrier in a bit period (higher frequency) and a binary “1” is defined as a 90° phase advance from the carrier in a bit period (lower frequency).

MSK MODULATOR

MSK signal generation is typically done by generating binary coherent FSK frequencies first. These waveforms are combined to produce the final signal s(t). Here is a block diagram of a typical MSK modulator:

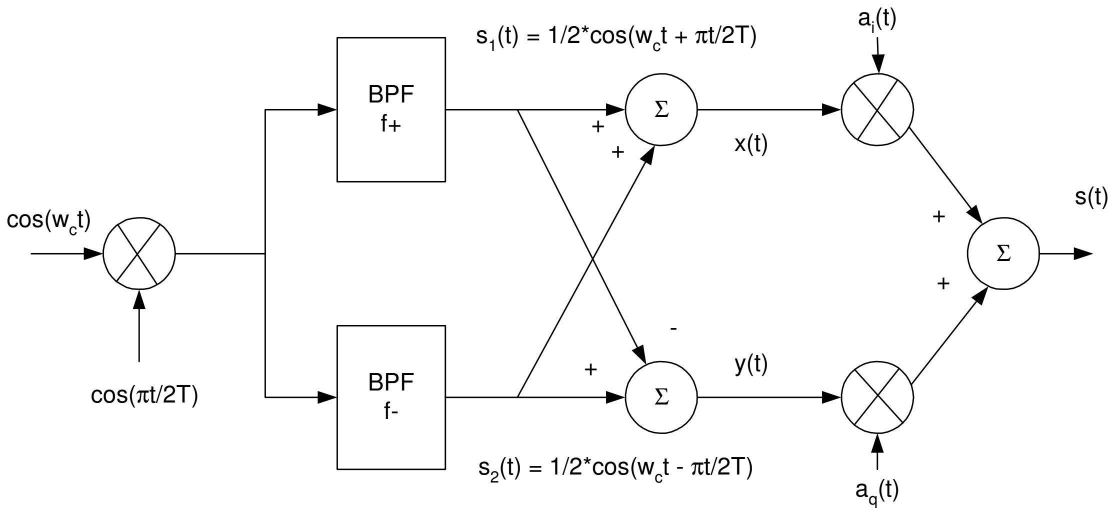


**Fig. 4 - MSK modulator**

Figure 4 demonstrates MSK signal generation via FSK. The signals are combined to form the proper x(t) and y(t) which are finally multiplied by the staggered bit sequences.

$$x(t) = \frac{1}{2}\cos(w_c t + \frac{\pi t}{2T}) + \frac{1}{2}\cos(w_c t - \frac{\pi t}{2T})$$
$$\therefore\ x(t) = \cos(\frac{\pi t}{2T})\cos(w_c t) \qquad (5)$$

$$y(t) = \frac{1}{2}\cos(w_c t - \frac{\pi t}{2T}) - \frac{1}{2}\cos(w_c t + \frac{\pi t}{2T})$$
$$\therefore\ y(t) = \sin(\frac{\pi t}{2T})\sin(w_c t) \qquad (6)$$

From Eq. 5 and 6, the signals x(t)+y(t)=s(t), as given by Eq. 3. This produces the proper MSK sequence that is phase continuous when the bit sequences are staggered by T seconds.

MSK RECEIVER

MSK can be demodulated using several techniques. Since MSK can be viewed as CPFSK, non-coherent and coherent demodulation is possible. Typical receivers will use a coherent demodulator with matched filtering to decode the incoming bit stream. The receiver architecture will be discussed later with respect to the beacon code.

To identify an MSK signal before demodulation, one can square the incoming signal. The squarer produces strong spectral components at double the frequency, specifically:

$$
\begin{aligned}
f_{+} &= 2(f_c + \frac{1}{4T}) \\
f_{-} &= 2(f_c - \frac{1}{4T})
\end{aligned} \tag{7}
$$

This signal is also known as Sunde's FSK. The frequency separation in this squared signal is now the bit rate 1/T. If the carrier is first removed from the signal before the squaring operation, then one would observe 2 strong peaks at DC separated in frequency by the bit rate, $\Delta f = 1/T$.